Notes on Laser Resonators

1 He-Ne Resonator Modes

The mirrors that make up the laser cavity essentially form a reflecting waveguide. A stability diagram that will be covered in lecture is shown in Figure 1.

\[ 0 < \left( 1 + \frac{d}{R_1} \right) \left( 1 + \frac{d}{R_2} \right) \leq 1 \]

Figure 1: Stability Diagram for standing-wave modes.

The solutions to the Maxwell equations, given a combination of spherical mirrors, results in several modes. The TEM modes are the transverse electric and transverse magnetic modes that resonate in the cavity. The lowest order mode approximates a Gaussian profile. A plot of these modes is shown in Figure 2.
1.1 Modes

Many applications of lasers require a minimum spot size. This requires that (a) the wavefront of the output has the same phase across its entire surface, and that (b) the edge of the wavefront falls off slowly, rather than abruptly as if it has passed through an aperture.

The TEM$_{00}$ mode in Figure 3 satisfies the two conditions mentioned above, while the mode has two lobes and thus does not satisfy requirement (a). The TEM$_{01}$ mode is a superposition of two degenerate TEM$_{01}$ modes - one with a sine dependence on the azimuthal angle and one with a cosine dependence. The sum of these two modes produces a “donut mode” with no field in the center of the mode. This mode has a spot size that is larger than that of the TEM$_{00}$ mode (see Reference 2).
1.2 Brewster Windows

Many laser lines including the 0.6328 μm for an He-Ne line have very low gain. As a result the laser cavity must be tailored to minimize losses.

![Figure 4: Configuration of Brewster Windows.](image)

Brewster windows (see Figure 4) are utilized at both ends of the laser tube to minimize reflection losses. As drawn in the figure, the Brewster windows will transmit vertically polarized light with no surface reflections. Consequently, the laser output will be polarized. For a He-Ne laser, other laser lines (1.15 μm and 3.39 μm) can be used by constructing cavities with appropriately coated mirrors, and different Brewster window material, as windows made of glass or quartz absorb infrared radiation and will thus increase the lasing threshold for these wavelengths.

1.3 Types of Resonators

For laser resonators, the only way for a uniphase wavefront to occur is to force all modes except for the lowest order mode TEM$_{00}$ to have high losses, so that they operate below threshold. To achieve this condition, the width of the lowest order mode should ideally equal the radius of the laser cavity mirrors. However, dust particles located on the mirror can produce different modal diffraction losses and can suppress the lowest order mode operation. It is therefore essential to design the laser with an adjustable mode waist $W_0$ to laser mirror aperture ratio. One way is accomplish this is to use an adjustable mirror separation. (see Figure 5) The beam waist $W(z)$ as a function of $z$ and the radius of curvature of the field $R(z)$ are derived in the book and in ECE 181. They are

$$W(z) = W_0 \left[ 1 + \left( \frac{\lambda z}{\pi W_0^2} \right) \right]^{1/2}$$  \hspace{1cm} (1)

$$R(z) = z \left[ 1 + \left( \frac{\pi W_0^2}{\lambda z} \right)^2 \right]$$  \hspace{1cm} (2)
Resonator Configurations

1. Plane-parallel resonator
   This was used in early designs of gas lasers. However, due to diffractions around the edges, the output has a diverging wavefront. Also, the ratio of TEM$_{01}$ diffraction loss to that of TEM$_{00}$ is a factor of two, which is much smaller than that of other resonator configurations. This small difference can easily lead to higher-order mode operation if other imperfections exist. Furthermore, a plane-parallel resonator is extremely susceptible to high losses resulting from mirror misalignment, mechanical vibrations, and thermal effects. It is also sensitive to the optical quality of the mirrors and the Brewster windows if they are used. For these reasons, plane parallel resonators are rarely used.

2. Large radius resonator
   For large $R$, we can neglect the factor of one in Eq.(2) and set $z = d/2$ so that the origin in halfway between the mirrors, we then have
   \[
   R \approx \frac{2\pi^2 W_0^4}{\lambda^2 d}
   \]
   so that
   \[
   W_0^4 \approx \left(\frac{\lambda}{\pi}\right)^2 \frac{d}{2} R
   \]
   Thus the waist $W_0$ is a very slowly varying function of $R$. Resonators with large radius mirrors can maximize the use of excited atoms in the cavity and thus potentially give high output power. Also, mirror alignment is not as critical as that in a plane-parallel resonator. We use large radius mirrors in the lab.

3. Confocal resonators
   When $R = d$, the resonator becomes confocal. From Eq.(2), $W_0$ is given by
   \[
   W_0 = \sqrt{\frac{R\lambda}{\sqrt{2}\pi}}
   \]
   Thus for a given distance between the mirrors $d$ or size of mirror $R$, this corresponds to the smallest $W_0$. As a result, confocal resonators are used whenever the smallest plasma tube diameter is desired, e.g. for $R = 300$ cm, $\lambda = 0.6328\mu$m, $2W_0 \approx 1.5$ mm. When $d$ is very close to $R$, $W_0$ varies very slowly with $d$. One disadvantage for confocal resonators
is that for two mirrors which have slightly different radii, say $R_1 < R_2$, then the resonator is stable only if $d < R_1$ or $d > R_2$. Thus in practice, $d$ has to be adjustable.

4. **Spherical resonator**
   When $2R = d$, it is called a spherical resonator. From Eq.(2), $W(z)$ is very large and it is focused down to a diffraction limited point at the center of the sphere.

### 1.4 Alignment of Resonators

The sensitivity to alignment variations of resonators can be deduced from the fact that the uniphase wavefront mode must lie symmetrically along the line joining the center of curvature of the two mirrors.

![Figure 6: Mirror alignment parameters. The parameter $b$ is the mirror radius $R$.](image)

In Figure 6, mirror $M_2$ is misaligned with respect to mirror $M_1$ by an angle $\theta$ measured from the center of $M_1$ with respect to the optical axis. The misalignment can also be measured from the center $C_2$ of the mirror $M_2$ with an angle $\phi$. Using the distances shown in Figure 6, we can then write

$$R_1 \theta = (R_1 + R_2 - d) \phi.$$  \hspace{1cm} (5)

Defining $\Delta x$, as the transverse displacement of the beam spot at $M_1$, and $\Delta y$ as transverse displacement of the beam spot at $M_2$, we have

$$\Delta x = x \phi = \frac{(R_2 - d) R_1 \theta}{R_1 + R_2 - d}$$  \hspace{1cm} (6)

$$\Delta y = y \phi = \frac{R_2 R_1 \theta}{R_1 + R_2 - d}$$  \hspace{1cm} (7)

where $x$ and $y$ are measured from the left side of the figure. The alignment sensitivity is then defined as the smaller of the two angles

$$\theta' = \frac{W_x \theta}{\Delta x} \text{ or } \theta' = \frac{W_y \theta}{\Delta y}.$$  \hspace{1cm} (8)

where the waist $W$ depends on the type of resonator. Small values of $\theta'$ are more sensitive.

With Eqs. (5), (6) and (7), we have the following summary:

1. **Large radius** ($R \gg d$): $\Delta x \approx \Delta y = R \theta / 2$ and $\theta' \approx 2W / R$. The alignment is not as critical as in (1).
2. **Confocal** $\Delta x = 0$, and $\Delta y = R \theta$ and $\theta'_y = W / R$. The alignment is least sensitive among all the various configurations.
3. **Spherical** $\theta' = W \delta / R^2$, where $\delta = R_1 + R_2 - d$ is small. The alignment is critical because $\delta$ is small.
2 Resonator Based Interferometers

Building a resonator and controlling the distance between the mirrors, one can vary the resonance frequency and thus construct a scanning interferometer. Scanning spherical-mirror interferometers (SSMI) are common tools for high resolution analysis of laser mode structures. They can be used in conjunction with a radio frequency (RF) spectrum analyzer which displays the frequency difference generated by the mixing laser modes when a square-law sensor is used. There are two types of laser modes. Longitudinal modes are associated with different modes of oscillation of the laser and are characterized by their oscillation frequency. Transverse modes are characterized by the field intensity distribution in a plane perpendicular to the direction of propagation. Several longitudinal modes can have the same form of the transverse field distribution. The SSMI are useful in examining the laser mode structure. Among the various SSMI, the mode degenerate interferometers, especially the confocal ones, are comparatively easy to use.

2.1 Scanning Fabry-Perot Interferometer

SSMI belong to the general class of Fabry-Perot interferometers, which consist of two mirrors placed parallel to each other with a separation \( d \). The resonance condition for such an interferometer is when the optical path between the mirrors is equal to an integral number \((m)\) of half wavelengths of the incident light. For normal incidence, the resonance condition is

\[
f_m = m \frac{c}{2d}
\]

where \( m \) is an integer and \( c/2d \) is the resonators free-spectral range. Let \( \phi = kd = \frac{2\pi mf d}{c} \) be the phase shift the field experiences where \( k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} \) is the wavenumber. The transmittance of the Fabry-Perot is specified by the Airy function

\[
\frac{I_{out}}{I_{in}} = \frac{1}{1 + \left(\frac{R}{1-R}\right)^2 \sin^2 (\phi)}
\]

where \( I_{in} \) and \( I_{out} \) are the input and output intensities, \( R \) and \( T \) are the reflection and transmission coefficient of the mirrors used in the resonator respectively. When \( R \approx 1 \), then the resonance features become very sharp. The Airy function near one of the resonance features \( \phi_m = \frac{2\pi mf d}{c} \) may then be written as

\[
\frac{I_{out}}{I_{in}} = \frac{1}{1 + \left(\frac{4}{1-R}\right)^2 (\phi - \phi_m)^2}
\]

which has the form of a Lorentzian with a full-width half-maximum width (FWHM) given by

\[
\Delta f = \frac{c(1 - R)}{2\pi d}
\]

This width specifies the instrument resolution.

The ratio of free-spectral range to \( \Delta f \) is called the finesse

\[
F = \frac{\pi}{c(1-R) \Delta f} = \frac{\pi}{(1-R)}
\]

An ordinary Fabry-Perot is not useful in analyzing laser modes. A typical gas laser transition has a Doppler linewidth of a few gigahertz, and a mode spacing of a few tens of Mhz for
the size of the resonators used in the lab. This implies that the interferometer must have a FSR larger than the Doppler linewidth, and a $\Delta f$ smaller than the mode spacing. For gas lasers, this typically means a finesse of at least 100. However, the finesse of an ordinary Fabry-Perot is limited by the flatness and the apertures of the plane mirrors, and for small apertures, diffraction loss becomes significant. A plot of a single resonance of a Fabry-Perot for three values of the finesse is shown in Figure 7.

![Plot of a single resonance feature of a Fabry-Perot for three values of the finesse.](image)

**Figure 7**: Plot of a single resonance feature of a Fabry-Perot for three values of the finesse.

### 2.2 Spherical Mirror Resonators

The diffraction effects of the Fabry-Perot etalon can be eliminated by using spherical, instead of plane, mirrors in the interferometer. However, the radius of curvature $R$ must be greater than $d/2$. The spherical mirror also alleviates the constraint on surface figure, because only a small area of the mirror is used. One requirement of a general spherical-mirror interferometer is that it must be illuminated with a narrow, diffraction limited beam, i.e. the beam must be mode-matched with the interferometer for proper operation. The various resonant modes of such a cavity is given by

$$f_{qmn} = \frac{c}{2d} \left[ q + \frac{1}{\pi} (1 + m + n) \cos^{-1} \left( 1 - \frac{d}{R} \right) \right]$$

where $q$ is an integer denoting longitudinal mode number, $m$ and $n$ are integers denoting transverse mode numbers. From Eq.(12), it can be seen that in order to have a large enough FSR, $m$ and $n$ must be fixed such that $\text{FSR} = c/2d$. In practice, the only transverse mode of the interferometer, which is convenient to excite is the TEM$_{00}$ mode. This single transverse mode requirement limits the use of a general spherical-mirror interferometer, as the laser itself may not operate in the TEM$_{00}$ mode. Moreover, an optical isolator must be used to avoid feedback of light from a well aligned interferometer into the laser.

### 2.3 Mode Degenerate Interferometers

In contrast to the general spherical-mirror interferometer, the mode-degenerate interferometer does not need to be mode-matched to the incident laser beam. This results in several simplifications:

1. The laser does not have to operate in a single transverse mode.
2. There is no spurious resonances due to mode-mismatching.
3. The interferometer need not be accurately aligned along the axis of the incident laser beam. This also allows the operation without an optical isolator as the light does not reflect back into the laser.

A mode-degenerate interferometer is a spherical-mirror interferometer whose transverse modes are degenerate in frequency. When the condition

\[
\cos^{-1}(1 - d/R) = \frac{\pi}{\ell}
\]

(where \( \ell \) is an integer) is satisfied, then the resonance condition in Eq.(12) becomes

\[
f_{qmn} = \frac{c}{2d} \left[ q + \frac{1}{\ell} (1 + m + n) \frac{\pi}{\ell} \right] = \frac{c}{2\ell d} [\ell q + (1 + m + n)]
\]

By increasing \( m - fn \) by \( \ell \), and decreasing \( q \) by 1, \( f_{qmn} \) remains unchanged. Thus the interferometer will have \( \ell \) "sets" of degenerate transverse modes with equal mode spacing. It has an FSR of \( c/2\ell d \) and a finesse of \( \pi/\ell (1 - R) \).

The best known mode-degenerate interferometer is the confocal one, with \( \ell = 2 \). It has "even symmetric" modes corresponding to \( (m + n) \) even, and "odd symmetric" modes corresponding to \( (m + n) \) odd. Mode degenerate interferometers can also be analyzed in terms of geometric optics. For instance, Eq.(13) is equivalent to the condition that a ray launched in the cavity retraces its path after \( \ell \) complete travels of the cavity. A typical ray path for the confocal interferometer is shown in Figure 8. For this resonator, \( \ell = 2 \) and the ray retraces its path after two complete round trips in the resonator regardless of the initial launch angle. This independence of round-trip on launch angle is the ray optics interpretation of mode degeneracy.

![Figure 8: Ray paths in a confocal resonator.](image)

The performance of mode-degenerate interferometers is limited by the mirror reflectivity, mirror surface figure, and spherical aberration. The reflectivity limitation is not serious as high reflectance (\( R > 0.998 \)) has been achieved in practice giving a finesse greater than 750. In order to minimize the effect of surface figure of the mirrors, the laser beam diameter should be as small as possible (approximately that of the TEM\(_{00}\) mode of the interferometer). To avoid aberration, the beam should be close to the interferometer axis, so that the paraxial optics approximation applies.

### 2.4 Practical Mode Degenerate Interferometers

Figure 9 shows a typical scanning confocal interferometer. It consists of two spherical mirrors, separated by a distance equal to their radius of curvature. The back surfaces of the mirrors are made such that the mirrors are self-collimating, so that a plane wavefront incident on the interferometer is transformed into a spherical wavefront with a radius of curvature that is
matched to the transverse modes of the interferometer. The concave surfaces are coated with high reflectance dielectric films; the convex ones are coated with anti-reflection films to eliminate spurious resonances associated with the back surfaces. The mirrors are mounted in a cell whose spacing can be controlled (to within a few wavelengths) by a voltage applied to the piezoelectric spacer. For confocal interferometers, a scan of $\lambda/3$ in spacing corresponds to one FSR. An aperture is placed outside the entrance mirror to limit the diameter of the incident beam and hence reduce spherical aberration. The transmitted light is detected by a photodetector whose electrical output is viewed on an oscilloscope, as a function of the voltage applied to the piezoelectric spacer.

It is important that the mirror separation be quite close to confocal. The tolerance on the mirror spacing depends on the length of the interferometer and its finesse. For very short, high finesse interferometers, the mirror spacing should be set to within a few wavelengths. In practice, the adjustment of length is quite easy to make. The mirror spacing should first be set so that it is approximately confocal; then by observing the mode structure of a laser on an oscilloscope, the final adjustment of the mirror spacing can be made to maximize the finesse of the interferometer. Once the separation of the interferometer mirrors is set, there is no need for further adjustment.

2.4.1 Use of Scanning Confocal Interferometers

Confocal resonators are useful for a variety of laser diagnostics. By focusing the laser beam into the scanning confocal interferometer, different transverse laser modes can be monitored, and the laser can be adjusted to operate in a single transverse mode. Fig. 4 shows the mode spectra for a He-Ne laser in double transverse mode, and single transverse mode operation.
2.5 Laboratory Scanning Interferometer

The Tropel model 240 used in the laboratory has relatively high reflectance dielectric mirrors. These mirror provide a good compromise between high spectral resolution and high instrumental transmission at the center of the bandpass. The exact separation of the two mirrors has been made insensitive to reasonable room temperature variations by the use of thermal compensation in the interferometer assembly.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
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<tr>
<td>Free spectral range</td>
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<tr>
<td>Instrumental bandwidth</td>
<td>$\Delta f$</td>
<td>75MHz</td>
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<tr>
<td>Spectral Resolving Power</td>
<td>$Q$</td>
<td>$8 \times 10^7$</td>
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<td>Finesse</td>
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<td>200</td>
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<tr>
<td>Peak instrumental transmission</td>
<td></td>
<td>20-30%</td>
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</table>

Table 1: Specification for Tropel 240 Interferometer.

We will used the Tropel 240 in the scanning mode shown in Figure 11.

Figure 11: Set-up for scanning interferometer.
2.5.1 Arrangement for scanning.

When used in the scanning mode, a time-varying voltage is applied to the "scanning voltage" terminal. This causes the mirror separation to change through the action of a piezoelectric transducer, and this in turn varies the resonant frequency of the interferometer. The model 240 will scan over one free spectral range, or 1500MHz, with the application of 30-40 volts. If the time-varying voltage is made periodic, the resultant repetitive display of the spectrum, with a 1500MHz scan period, provides a convenient self-calibration of the frequency scale. Note that an increase in applied scan voltage produces a decrease in the resonant frequency of the interferometer. Figure 12 illustrates a typical oscilloscope trace obtained when the model 240 is used in the scanning mode.

There are a few points to consider when using the model 240:

1. If the spectrum analyzer and laser beam are exactly co-linear, the spectrum of the laser may become an erratic function of time due to optical feedback from the spectrum analyzer into the laser. This can be eliminated by a slight adjustment of the alignment, or by the insertion of a circular polarizer (linear polarizer plus 1/4-wave plate) between the laser and the spectrum analyzer. A relatively slow temporal variation of the spectrum of the laser is typical of many commercially available lasers and is due to thermal variations in the laser cavity length.

2. Once a high resolution display is obtained on the scope, the user should experiment to find out the angle through which the alignment can be varied without seriously affecting the instrumental bandwidth. In general, the final alignment is accomplished by observing the displayed spectrum while touching up the alignment adjustment screws to maximize the amplitude of the display, and to minimize the linewidth of each observed longitudinal mode.

3. Whenever possible, use a collimated beam with a small beam diameter (on the order of 1-2mm). This will provide the maximum signal and minimum instrumental bandwidth, while at the same time will allow maximum alignment tolerance.

Figure 12: Typical laser spectra.
4. Scan rates greater than a 200Hz should be avoided because the scan becomes nonlinear at high frequencies. Moreover, the silicon photo diode cannot resolve pulses shorter than a few microseconds, since zero reverse bias voltage is applied to the detector in the model 240. For continuous scans, a sinusoidal scanning voltage should be used to prevent damage to the piezoelectric transducers. In practice, a scanning voltage of around 35Vp-p at 100 Hz is recommended for obtaining an initial display on the scope. sinusoidal scanning voltage input

3 References

These notes are abstracted from:


